22-23/12154

M.Sc. 1st Semester Examination-2022-23

MATHEMATICS

Course ID : 12154 **Course Code : MATH/104C**

Course Title : Ordinary Differential Equations and Partial Differential Equations

Time : 2 Hours *Full Marks* : 40

The figures in the right hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer **any *five*** questions : 8×5=40

1. (a) State the basic existence theorem for Cauchy problem.

(b) Find the parallelogram identity for the wave equation when the wave speed $c \neq 1$.

(c) Find the adjoint of the following PDE:

$$u_{xx} + 4u_{xy} + u_x = 0.$$

(d) State the Picard's theorem. 2+2+2+2

2. Solve the following D'Alembert's problem 8

$$u_{tt} - u_{xx} = xt, \quad -\infty < x < \infty, \ t \geq 0$$

$$u(x, 0) = 0, \quad -\infty < x < \infty$$

$$u_t(x, 0) = e^x, \quad -\infty < x < \infty .$$

3. Solve using the seperation of variables 8

$$u_t(x, t) = k \, u_{xx}(x, t), \quad a < x < b, \ t > 0$$

with $u(x, 0) = f(x)$

$u(a, t) = u(b, t) = 0$, k, a and b are constants.

4. Consider the linear differential equation

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

where P, Q are either constants or functions of x alone. Prove that two solutions of this equation are linearly dependent if and only if their Wronskian vanishes identically. 4+4

5. Construct Green's function to solve the boundary value problem 8

$$\frac{d^2y}{dx^2} + 4y = f(x), \quad y(0) = 0, \ y\left(\frac{\pi}{4}\right) = 0$$

6. Find a complete integral of [illegible] 6

7. Prove that the Eigen functions of a regular Sturm-Liouville problem corresponding to distinct eigenvalues are orthogonal w.r.t. weight function $r(x)$ on $[a, b]$, that is, if $y(x)$ and $z(x)$ are Eigen functions corresponding to distinct eigenvalues λ and μ respectively, then

$$\int_a^b r(x)y(x)z(x)\,dx = 0.$$ 6

8. Solve $\frac{d^2y}{dx^2} + \lambda y = 0$, where [illegible] 6

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